The various contours and boundaries discussed previously are illustrated in Fig. 2. For completeness, the CEP and the projection of the SEP are also shown on the diagram. The example shows the considerable variation possible for different contours associated with the same covariance matrix. The analyst, therefore, must carefully consider the alternatives and their interpretations before making a choice. While in general, there is no "best" geometric interpretation of a Gaussian probability distribution, error ellipses and ellipsoids are usually preferable since they contain more information than the other forms.

References

[1] Gura, I. A. and Gersten, R. H., "On Analysis of n-Dimensional Normal Probabilities," TR-0066(5129-01)-2, June 2, 1970, Aerospace Corp., El Segundo, Calif.

[2] Greenwalt, C. R. and Shultz, M. E., "Principles of Error Theory and Cartographic Applications," Rept. 96, Feb. 1962, Aeronautical Chart and Information Center, St. Louis, Mo.

[3] Schulte, R. J., "Four Methods of Solving the Spherical Error Probable Associated with a Three-Dimensional Normal Distribution," AD 666646, Jan. 1968, Holloman Air Force Base, N. Mex.

Effects of Prestress on Vibration Behavior of Certain Shells of Revolution

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Nomenclature

E = modulus of elasticity
h = shell thickness
n = circumferential wave number
p = external pressure
p_0 = reference buckling pressure = $[2Eh^2/R^2][3(1 - \nu^2)]^{-1/2}$
R = spherical shell radius
α = half opening angle of shell
λ = shallowness parameter = $[12(1 - \nu^2)]^{1/4}[R\alpha^2/h]^{1/2}$
ν = Poisson's ratio
ρ = mass density
Ω = natural frequency
Ω_0 = reference frequency = $[E/\rho R^2(1 - \nu^2)]^{1/2}$

Introduction

The vibration and buckling behavior of shells of revolution have been studied extensively and numerous publications exist in the literature (see Refs. 1–6). However, data are lacking on the vibration behavior of prestressed shells which span the range between unstressed and buckled shells. These data are needed to provide a better understanding of the influence of prestress on vibration behavior and to provide results to which new analyses may be compared as they are developed. This Note presents the results of the vibration analysis of two prestressed shells of revolution obtained with the VALORS and BALORS programs which were early vibration and buckling versions of the SALORS (Stress Analysis of Layered Orthotropic Ring—Stiffened Shells of Revolution) System described in Ref. 7.

Shallow Spherical Cap

The clamped aluminum shallow spherical cap shown in Fig. 1 has a radius of 100 in., a half-opening angle of 20°, and a thickness of 1.094 in.; Young's modulus is 10^7 psi, and Poisson's ratio is $\frac{1}{3}$. The shallow shell parameter λ is 6. The shell is subjected to a uniform external pressure. The first-mode vibration behavior, for n = 0–4, was studied under the action of both linear and nonlinear prestress including prestress rotation. Initially the first natural frequencies of the unstressed shell were determined; resulting values of the frequency parameter Ω/Ω_0 are 1.4984, 1.1730, 1.3425, 1.6522, and 2.0629 for n = 0, 1, ..., 4, respectively, which compare favorably with frequencies for a similar shell in Ref. 8.

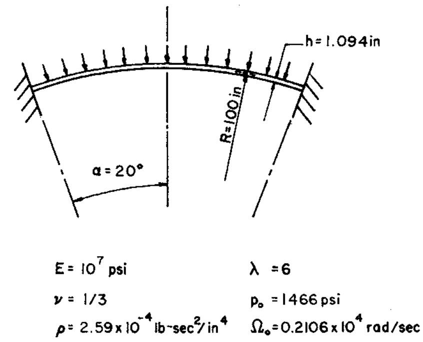

Fig. 1 Shallow spherical cap.

Figure 2 shows fundamental frequency vs prestress for several values of n and for both the linear and nonlinear prestress solutions. The pressures corresponding to zero frequency for the nonlinear prestress results agree with the buckling loads given by Huang;[9] those for the linear prestress agree with buckling loads obtained with the BALORS program and results given in Ref. 4. The results show the strong influence of the prestress nonlinearity on the frequencies of the shell as the load nears the buckling loads.

Conical Sandwich Shell

Figure 3 shows the conical sandwich shell. Properties are indicated clamped at the small end, free at the ring-stiffened large end, and subjected to an external uniform pressure. The shell is covered by a heat shield with substantial mass but negligible stiffness. A stress analysis and some of the

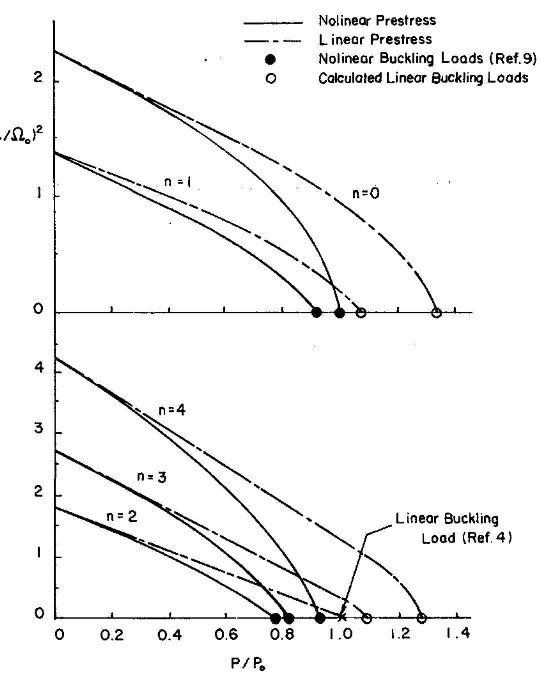

Fig. 2 Variation of natural frequencies of spherical cap with linear and nonlinear prestress (n = 0, 1, and 2,3,4).

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unstressed natural frequencies of this shell have previously been reported.[10]

The prestressed vibration of this shell, using exact linear prestress together with prestress rotations was determined. The effect of the discrete ring at the boundary was obtained from the ring theory presented in Refs. 2 and 4 and includes both ring prestress and mass. Figure 4a shows the square of the first natural frequency vs external pressure for circumferential wave numbers from 0 to 7. The Ω^2 intercepts give the unstressed natural frequencies while the p intercepts the buckling pressures. Selected calculations were carried out with the BALORS programs to confirm the buckling loads indicated by the prestressed vibration results.

The results for n equal 0 and 1 indicate a hardening behavior (i.e., increased natural frequencies for the the first vibration modes) under increased loading rather than the typical softening behavior usually associated with prestressed vibrations. This behavior should be expected, since the lowest buckling loads associated with these modes ($n = 0, 1$) are negative i.e., they correspond to buckling due to an internal pressure. The results also indicate that the lowest buckling load occurs at n equal 6, which is typical of this class of shell.[11]

Since efficient design of conical aeroshells requires selection of the ring size on the basis of simultaneous failure in a ring buckling mode ($n = 2$) and a shell buckling mode ($n = 5,6,7$) ring prestress should play an important role in obtaining a properly designed shell. Neglecting the ring prestress will result in higher values for the computed natural frequencies. Figure 4b compares the exact results for low n with those obtained when the ring prestress is neglected. Neglecting the ring prestress can lead, for $n = 2$ and 3, to substantial differences in frequency when the load nears the respective buckling loads. The extreme sensitivity of buckling loads, for low n mode shapes, to ring stiffness, which is evident in these results, has been noted previously.[12]

Concluding Remarks

The prestressed vibration results for the shallow spherical cap indicate the importance of including the effects of the nonlinear prestress state in the determination of the shell's natural frequencies. Linear interpolation between an unstressed natural frequency and the corresponding nonlinear buckling load leads to substantial error in frequency for stresses near the buckling load.

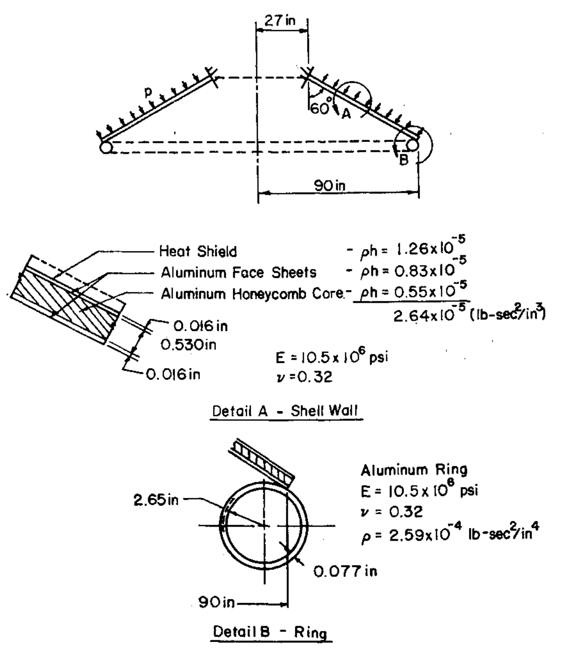

Fig. 3 Conical sandwich shell.

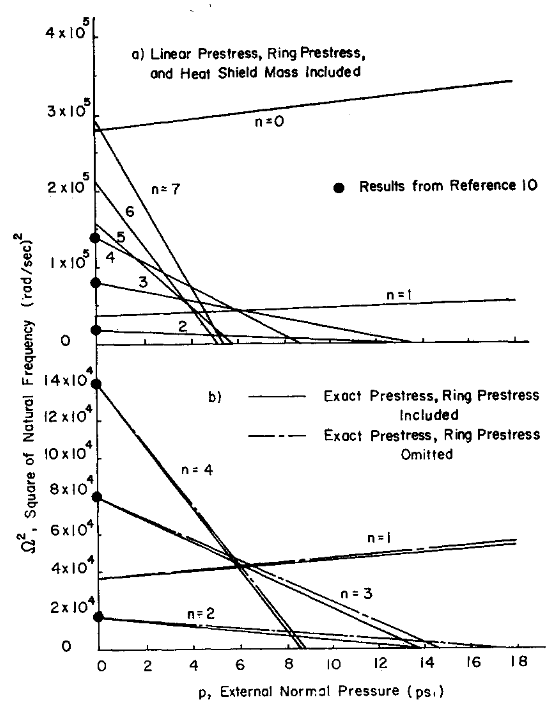

Fig. 4 Effects of prestress on natural frequencies of conical shell.

For the conical shell, the $n = 2$ and 3 modes are strongly influenced by the ring boundary conditions, and proper account of ring prestress is necessary. In addition, it is seen that, depending on the circumferential wave number, prestress can have either a softening or hardening effect on frequency.

References

[1] Kalnins, A., "Free Vibration of Rotationally Symmetric Shells," *The Journal of the Acoustical Society of America,* Vol. 36, 1964, pp. 1355–1365.

[2] Cohen, G. A., "Computer Analysis of Asymmetric Free Vibrations of Ring-Stiffened Orthotropic Shells of Revolution," *AIAA Journal,* Vol. 3, No. 12, Dec. 1965, pp. 2305–2312.

[3] Cooper, P. A., "Vibration and Buckling of Prestressed Shells of Revolution," TN D-3831, 1967, NASA.

[4] Cohen, G. A., "Computer Analysis of Asymmetric Buckling of Ring-Stiffened Orthotropic Shells of Revolution," *AIAA Journal,* Vol. 6, No. 1, Jan. 1968, pp. 141–149.

[5] Block, David L., "Influence of Discrete Ring Stiffeners and Prebuckling Deformations on the Buckling of Eccentrically Stiffened Orthotropic Cylinders," TN D-4283, 1968, NASA.

[6] Almroth, B. O., Bushnell, D., and Sobel, L. H., "Buckling of Shells of Revolution With Various Wall Constructions," CR-1049, 1050, and 1051, Vols. 1, 2, and 3, 1968, NASA.

[7] Anderson, M. S., et al., "Stress, Buckling and Vibration Analysis of Shells of Revolution," Conference on Computer Oriented Analysis of Shell Structures, Palo Alto, Calif., Aug., 1970 (preprints available from NASA-Langley Research Center).

[8] Zarghamee, M. S. and Robinson, A. R., "A Numerical Method for Analysis of Free Vibration of Spherical Shells," *AIAA Journal,* Vol. 5, No. 7, July 1967, pp. 1256–1261.

[9] Huang, Nai-Chien, "Unsymmetrical Buckling of Shallow Spherical Shells," *AIAA Journal,* Vol. 1, No. 4, April 1963, p. 945.

[10] Cohen, G. A., Foster, R. M., and Schafer, M., "Analysis of Conceptual Designs for the Voyager Entry Capsule," CR-66580, NASA.

[11] Cohen, G. A., "The Effect of Edge Constraint on the Buckling of Sandwich and Ring Stiffened 120 Degree Conical Shells Subjected to External Pressure," CR-795, May 1967, NASA.

[12] Cohen, G. A., "Buckling of Axially Compressed Cylindrical Shells with Ring Stiffened Edges," *AIAA Journal,* Vol. 4, No. 10, Oct. 1966, pp. 1859–1862.